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Empowering Young Minds:

A Case Study of Student-Centered Mathematics in South Sudan

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Abstract:

The teaching and learning of mathematics in sub-Saharan African countries is dominated by teacher-centred pedagogies rather than student-centred ones. Observations of mathematics teachers at two private schools in South Sudan confirmed such practices. This inspired the researchers to design an intervention to help six primary mathematics teachers shift their practices through problem-solving and mathematical discourse. Design-based research methods were implemented, and data were gathered using observations supported by video and audio recordings and field notes. The participants were selected using convenience sampling, and the data were analysed using Stephan's checklist of student-centred teaching as a framework. The findings revealed that initially, teachers were using tasks from textbooks, and the teachers themselves were engaged in solving the tasks while their students reproduced their actions. Additionally, all the teachers dominated classroom discussions. After the intervention, the teachers began to select tasks that could enhance learning through problem-solving and mathematical discourse among the students, shifting the teacher's role to facilitation as the students engaged in solving the tasks on their own. However, the shift in practice was highly dependent on the intervention, as the teachers' tasks were adopted from the workshop.

Keywords: Student-centred teaching; teacher-centred teaching; problem-solving; mathematical discourse; teaching.

Introduction

South Sudan has been experiencing perplexing situations of untrained and unqualified teachers. Providing these teachers with professional learning during their in-service training is vital. To this end, there are different types of professional development (PD) programmes provided by the government and non-governmental organisations (NGOs) (Soforon et al., 2023). This study focuses on engaging mathematics teachers in South Sudan with a programme of training that is interwoven with effective PD (Soforon et al., 2023). According to Hunzicker (2011), effective PD programmes are supportive, job-embedded, instructional-focused, collaborative, and ongoing. Effective PD equips the teachers with new knowledge, skills, and the current trend approaches to teaching. The role of this training is to ensure that teachers are equipped with new approaches, strategies, and techniques for teaching, hence improving their students' performance in mathematics (Soforon et al., 2023).

South Sudan compares to other sub-Saharan African countries in allowing the teachers to take sole autonomy of both teaching and learning, thus rendering the students passive recipients in the classroom (Eltayeb-Abdalla & Nour-Alsiddiq, 2016). Student-centred teaching (SCT) is a system of instruction where the students are placed at the centre of both teaching and learning. The role of the teacher is to facilitate active participation and independent inquiry among students. Engaging mathematics teachers with SCT is the main theme of this research. As observed in other educational settings within sub-Saharan Africa (Anyanwu & Iwuamadi, 2015; Bethell, 2016, Van de Kuilen et al., 2019), teachers in South Sudan (Eltayeb-Abdalla & Nour-Alsiddiq, 2016) are heavily influenced by the practice of instrumental understanding. Teachers are more concerned with what they teach instead of what the students can construct as part of learning.

In order to bridge the gap between the South Sudan educational system and SCT, there has been a paradigm shift in the recent South Sudan National Curriculum where it accentuates the current implementation of the SCT approach to be used in all levels of education. Stephan (2014, p. 338) highlighted that:

… teachers are seen to be the authors of knowledge, skills, and wisdom regarding the teaching and learning aspects, where they are engaged directly in lecturing, solving every task for the students, and using step-by-step methods (procedures). This corresponds to the notion that teachers in this situation are the commanders-in-chief of learning where they control every aspect of the teaching and learning.

Stephan (2014, p. 339) reflected that 'since the publication of the National Council of Teachers of Mathematics Curriculum and Evaluation Standards for School Mathematics in 1989, there has been a significant push towards SCT in mathematics'. Since then, SCT has grown prominent in both research and teaching (Corkin et al., 2018; NCTM 2014; Stephan, 2014). The researchers propose introducing the SCT approach as the remedy to engage teachers in shifting their practices from teacher-centred to student-centred. Notably, SCT plays a substantial role in improving the teachers' practices, beliefs, and attitudes thus enhancing students' performance (Eltayeb-Abdalla & Nour-Alsiddiq, 2016; Weimer, 2002). Training teachers to shift from their initial approach of teaching to a more modern fashion of teaching is a demanding task (Corkin et al., 2018).

However, numerous literatures have demonstrated how teachers are engaged in using SCT to enhance quality of education and effective teaching in both sub-Saharan Africa and globally (NCTM, 2014; Stephan, 2014). However, in South Sudan, little is being done in this area according to the Ministry of General Education and Instruction (2017). Therefore, this gap has triggered the researchers to conduct critical investigation in this area, to establish the findings associated with this study. Stephan (2014) provided five characteristics that described the SCT classroom: a focus on problem-solving, classroom environment, collaboration, mathematical discourse, and tools or manipulatives. This article only focuses on two of these aspects, namely problem-solving and mathematical discourse. Hence, we ask the following research questions:

(1) To what extent do the primary mathematics teachers in two private schools in South Sudan engage the students in problem-solving to enhance SCT?; and

(2) To what extent do the primary mathematics teachers in two private schools in

South Sudan engage the students in mathematical discourse to enhance SCT?

Teacher-centred teaching versus student-centred teaching

Traditionally, school mathematics has been dominated by the practice of teacher-centred teaching (TCT) (NCTM, 2014; Stephan, 2014). The teachers do the mathematics, and the pupils reproduce what the teacher does (Fosnot & Dolk, 2001). Teacher-centred teaching 'is characterised by the traditional formula-based strategy that focuses on computation with little reference to mathematical reasoning and problem-solving' (Bature, 2020, p. 3). This instinctively occurs by ensuring that students are taught how to master the formula or algorithm and then practise and apply it to determine the solutions to the tasks. Such mathematical teaching and learning are associated with instrumental understanding (Skemp, 1987). Bature relates TCT to memorisation, recitation, imitation, and being procedural without meaningful conceptual understanding, and it is highly associated with the behavioural learning paradigm (Bature, 2020; NCTM, 1989). Thus, the students are in a state of total interdependence with their teachers during the teaching and learning process instead of undertaking the autonomous stance of both teaching and learning. This type of teaching is also referred to as direct instruction, deductive teaching, or expository teaching and is typified by the lecture-type presentation (Bature, 2020; Stephan, 2014).

On the other hand, SCT refers to an approach to education that focuses on the individual student's needs (Anyanwu & Iwuamadi, 2015; Bature, 2020; NCTM, 1989; Stephan, 2014; Walters et al., 2014; Weimer, 2002). This type of learning puts the students at the centre of the learning process and gives them more autonomy in what they are learning (NCTM, 1989; Stephan, 2014). In SCT, the teacher is more of a facilitator than a lecturer (Garrett, 2008; Hokor & Sedofia, 2021). Eltayeb-Abdalla and Nour-Alsiddiq (2016, p. 10) define SCT as:

> … a system of instruction that places the student at its center. It is teaching that facilitates active participation and independent inquiry and seeks to instill among students the joy of learning inside and outside the classroom.

Student-centred teaching allows the students to be seen as more extrinsically motivated and learn essential skills such as critical thinking and problem-solving among themselves with minimal assistance from their teachers (Emanet, 2021). The students brainstorm, interact, and dialogue among themselves as a means to expedite their acquisition of skills and knowledge (Polly and Hannafin, 2010). By considering the cultural context of SCT in South Sudan, the majority of the teachers declined to apply this new strategy due to their cultural norms, beliefs, and familiarity with TCT. There has been a clear reservation that the practical implementation of this approach may suffer due to factors such as limited resources, class size, teacher training, and administrative support, which can impact the feasibility and effectiveness of implementing student-centred approaches (Mueller et al., 2014; Mukuka et al., 2023). This SCT ensures that equity and inclusion are interwoven into problem-solving and mathematical discourse, aiming at boosting equitable access to quality education for all students, where marginalised or underrepresented backgrounds are considered. This strategy may include appropriate application of instruction, providing additional support for struggling students, and fostering inclusive classroom environments where all students feel valued and empowered to participate in the study.

Furthermore, it must be explicitly understood that SCT does not recognise students as passive recipients of information but as active agents engaging in constructing their own knowledge

(NCTM, 2014; Pathan et al., 2018). Weimer (2002, p. 57) discusses the:

… key changes necessary to shift students from surface learning to deep learning, which includes a shift from TCT to SCT by pointing to five components: (a) the balance of power, (b) the function of course content, (c) the role of the teacher, (d) who is responsible for learning, and (e) the purpose and process of evaluation.

These characteristics are general to STEM education (Walters et al., 2014; Weimer, 2002).

Walters and colleagues (2014) studied high school teachers and their students, providing a framework that proposes the characteristics of a student-centred mathematics teaching with two broad categories: classroom environment and mathematics instruction. First, the classroom environment should be supportive. It means it should be respectful, provide a strong relationship, and focus on the individual (scaffolding, differentiation, and choice). The mathematics instruction should provide students with opportunities to: use mathematical reasoning, communicate their mathematical thinking, and critique the reasoning of others; make connections; and solve mathematical problems (Mueller et al., 2014; Mukuka et al., 2023; Walters et al., 2014). Indulging in problem-solving or being exposed to problem-solving affords students the opportunity to engage in high-level thinking (NCTM; 2014; Stephan, 2014). Therefore, teachers must regularly select and implement tasks that promote reasoning and problem-solving (NCTM, 2014). Tasks that encourage reasoning and access to mathematics through multiple entry points, including the use of different representations and tools, can foster the solving of problems through varied solution strategies (Boaler, 2016; NCTM, 2014; Stephan, 2014).

Stephan (2014) proposed a framework with five characteristics: problem-solving (PS), classroom environment (CE), collaboration (Col), mathematical discourse (MD), and manipulatives or tools (MT) to foster SCT. These are embedded in the NCTM's (2014) effective mathematics teaching practices. In this study, we only focus on a critical investigation of how PS and MD help to shift the TCT practice of six mathematics teachers in two South Sudan school settings to SCT. The remaining three aspects, that is to say, CE, CO, and MT, are deferred for another publication. Hence, PS and MD are presented further.

Problem-solving

A problem is a task, situation, or activity students get from their teachers or face in life for which they do not have a ready-made formula or strategy to solve (Cai et al., 2015; Hiebert et al., 1996; Kilpatrick, 1987; Schoenfeld, 1992). Avcu and Avcu (2010, p.1282) defined a problem as 'a situation that one faces with some blockage while solving the problem'. Nurkaeti (2018) referred to PS as a task where the students are engaged in solving either routine or non-routine problems. Routine problems are mostly seen in textbooks and can be solved through basic operations. For instance, a routine problem is a kind of mathematical task that is closed-ended. This kind of problem or task often uses an algorithm or formula to arrive at the desired solution for the problem in question. Non-routine problems require planning, organising, and classifying data, discovering the relations, and determining the rules and generalities. Usually, a non-routine problem has no predetermined algorithm or formula or method to be applied in solving the given task. Precisely, this is an open-ended task that requires conceptual understanding, internalising, and integrating conceptual reasoning into the real-world situation. Rich and open-ended tasks can provide opportunities for rich learning by engaging students in PS and mathematical thinking (Boaler, 2016; Mueller et al., 2014; NCTM, 2014).

Many researchers have advocated that problem posing and PS are central to mathematical thinking, creativity, and discourse in mathematics (Cai et al., 2015; Hiebert et al., 1996; Silver, 1994; Tesfamicael et al., 2020). Problem-solving refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development (Cai et al., 2015).

Van de Walle et al. (2020) presented three approaches to PS which were described by Schroeder and Lester (1989): teaching for PS, teaching about PS, and teaching through PS. 'Teaching for PS starts with learning the abstract concept and then moving to solving problems as a way to apply the learned skills (explain-practice-apply)' (Schroeder & Lester, 1989, p. 55). Teaching for PS follows traditional mathematics teaching, where mathematics rules and formulas are applied in word or text problems while teaching about PS is about providing guidance to students to solve problems. George Polya (1945) proposed four steps for PS: understanding the problem, devising a plan, carrying out the plan, and looking back. However, the third approach is of interest to this study. Teaching through PS is an approach where students learn mathematics through inquiry by exploring texts, problems, situations, patterns, and models (Boaler, 2016; Cai et al., 2015; Hiebert et al., 1996).

Teaching through PS typically engages the students to participate in deep thinking and conceptual reasoning as they encounter mathematical problems (Hiebert et al., 1996; NCTM, 2014; Stephan, 2014; McGatha et al., 2018). Teachers are expected to use tasks that lend themselves to multiple representations and strategies (McGatha et al., 2018). This is regarded as teaching through PS, where students ultimately grapple by themselves to find meaningful solutions to the mathematical problems posed by their teacher. Hence, students are engaged with the notion of solving mathematical problems by using critical thinking skills and reasoning to strengthen their solutions (Smith & Stein, 1998; Stephan, 2014).

Mathematical Discourse

Facilitating meaningful MD is one of the eight effective mathematics teaching practices promoted by NCTM (2014). Stephan (2014) considers MD as one of the crucial aspects of student-centred instruction that involves using student discourse in whole-class discussion to bring out important … mathematical ideas. Mathematical discourse refers to verbal and written communication that is centred around deepening, thinking about, and making sense of mathematics (Sfard, 2012). Through MD, students discuss, brainstorm, and engage in critical thinking and reasoning in pursuit of arriving at the answer to the problem (Ballard, 2017). Celik and Baki (2023) explain MD as a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts' of thinking, feeling, believing, valuing, and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network', or to signal (that one is playing) a socially meaningful role. This explanation regards MD as a means to unite students by dialoguing, agreeing, and disagreeing to establish consciousness over a critical matter requiring collegial understanding (Jill and Erlina, 2015).

In TCT classrooms, teachers stand at the front of the room and dominate the conversation (NCTM, 2014). According to Drageset (2015), such practice can be described by the Initiation Response Evaluation (IRE) model of discourse, which is a theoretical framework describing a discourse pattern where the teacher initiates the questions, the students respond to them, and the teacher evaluates the responses (Drageset, 2015; NCTM, 2014). In SCT classrooms, teachers should allow the students to explore various strategies and approaches, especially when encountering mathematical problems (Stephan, 2014). The students undertake the autonomy of calculating the mathematical tasks being posed to them by their teacher (Stephan, 2014). Contrarily, if the teacher undertakes full control of what is being said by the students, this can ensure that the lecture includes the intended mathematics goal. On the other hand, with student-led discussion, the teacher carefully guides the students toward discussing the intended mathematical tasks (Cobb, 1994; Stephan, 2014).

In general, leading an effective classroom discussion is a demanding task, and there are several research-based protocols that mathematics teachers can implement in their classrooms (Ballard, 2017; Faria et al., 2024; Hufferd-Ackles et al., 2004; Larsson, 2015; Smith & Stein, 2011). For instance, Hufferd-Ackles et al. (2004) underline four developmental trajectories in the Math-Talk Learning Community that consist of:

(1) questioning,
(2) explaining mathematical thinking,
(3) sources of: mathematical ideas, and
(4) responsibility for learning.

Chapin et al. (2009) provide talk moves and tools that can help facilitate discourse, including revoicing, repeating, reasoning, and adding on. Additionally, selecting open-ended, high-level thinking, and conceptually focused tasks or questions is vital for facilitating effective discourse in the classroom (McGatha et al., 2018). Smith and Stein (2011) provide a framework that can help orchestrate a productive classroom discussion. It has five elements: anticipating student responses to challenging mathematical tasks, monitoring students’ actual responses to the tasks, selecting particular students to present their mathematical work, sequencing and sharing students’ reactions in a specific order for discussion, and connecting different students’ responses to crucial mathematical ideas (Ballard, 2017; Faria et al., 2024; Larsson, 2015). These demand high competency on the part of the teachers to implement in the classroom.

For this study, part of the four-level NCTM (2014) framework, which was developed by Hufford-Ackles et al. (2014), is used to situate the six mathematics teachers’ practices before and after the intervention due to its simplicity in implementing it in the selected South Sudan school context. At Level 0, the teacher dominates the conversation. At Level 1, the teacher encourages the students to engage in discourse with the whole class. At Level 2, the teacher facilitates conversation between students and encourages them to ask one another questions. Finally, at Level 3, students carry the conversation themselves while the teacher guides them from the periphery (NCTM, 2014).

Methodology

Research design

This article emanated from a doctoral study focusing on SCT in the context of South Sudanese schools where the researcher adopted design-based research (DBR) as the methodological approach to guide the study. Design-based research refers to the systematic study of designing, developing, and evaluating educational interventions. According to Campanella and Penuel (2021) and Cobb et al. (2003), DBR helps to foster learning, create usable knowledge, and advance theories of learning and teaching in complex settings (Fowler et al., 2023). The rationale for using DBR as the research methodology is that it bridges the gap between theory and practice. Gravemeijer and Prediger (2019) provided five approaches regarding the concrete realisation of DBR: interventionist, theory generative, prospective, and reflective, iterative, and pragmatic roots, and humble theories.

This study employed theories about SCT and trained the six teachers after observing their teaching practice. The study identified how SCT can be intertwined with DBR and the practical application of the intervention to enhance effective teaching and learning of mathematics in South Sudan elementary schools. It is worth mentioning that intervention requires iteration of the intended programmes or tasks of SCT to satisfy the solutions to the problems identified with TCT. This occurred when the researcher observed the teachers and identified that they were using the TCT strategy and that this was a practical problem at hand. A remedy was designed by the researcher which was the application of the SCT approach. This SCT strategy was introduced to the teachers by the researcher through PD training. After successful training, the teachers were sent back to their respective schools and the researcher followed them and observed how they applied the strategy of SCT in their classrooms. Having completed the observation of these teachers, the researcher again invited them for reflection and the programme kept iterating cyclically. Thus, the reasons for using the intervention feature of DBR are as follows: (1) intervention often facilitates the process of multiple outcomes to be realised in the forms of better achievement, improved student attitudes, and increased teacher satisfaction, and (2) it helps to articulate the process of inputs by designing a conducive learning environment to promote both certain instructional learning materials and teacher development.

Figure 1 shows parts of the design elements and process in this study, indicating the operationalisation of the DBR as illustrated. Firstly, selecting schools and mathematics teachers to participate in the study was completed. After negotiating with educational officials, principals, and teachers in two schools, six teachers, three from each school, were identified. The two schools were conveniently chosen based on proximity, and the six teachers were those engaged in teaching mathematics at the Grade 5 level. So, convenient sampling was used to make access to the schools as simple as possible for the researcher. To design and implement context-based professional learning, the researcher conducted a preliminary study on types of PD and the meaning of effective PD in the South Sudan education system (Soforon et al., 2023). These activities define the pre-DBR phase of the study. Following that, four days of intervention on SCT were planned and conducted as a workshop. The researcher then observed the teachers in their classrooms to assess their actions and collect their reflections. Subsequently, the researcher observed them in their classes again.

Data collection

Instruments such as interviews, observation, recorders, cameras, and field notes were used for collecting the data. However, they were regarded as tools for social interaction in the conversation (Rubin & Rubin, 2012). The interviews held with the six teachers before and after the intervention formed part of the data. The researcher observed the teachers while teaching, depending on their classroom schedules. Furthermore, audio and video were recorded and the lessons photographed using both recorder and camera during the classroom observations. Lastly, the field notes were compiled for documenting or detailing the information written by the teachers on the chalkboard as the teaching was ongoing in the respective classrooms.

Selection of two schools to participate in professional development

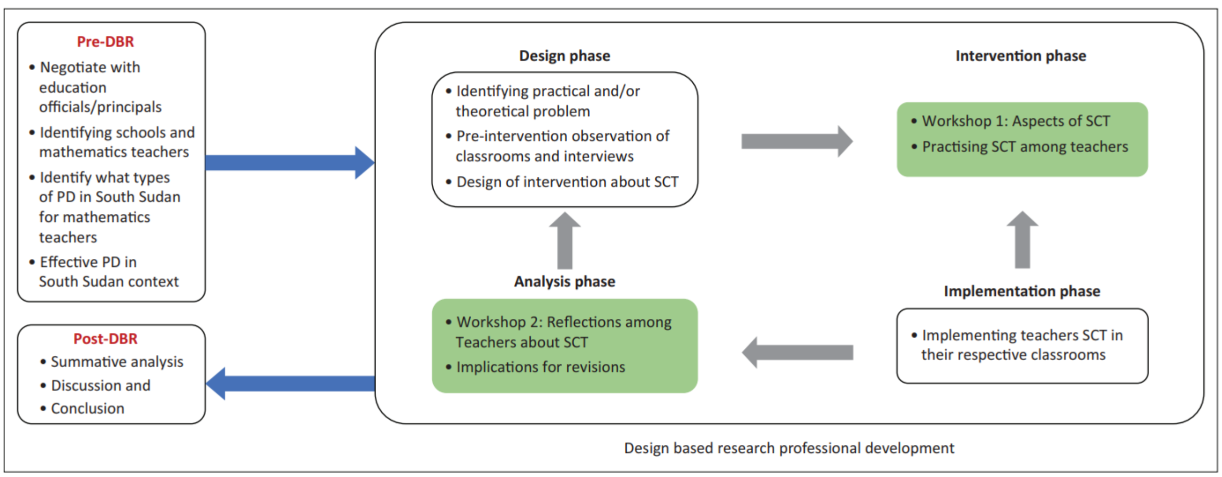
Two primary schools, School 1 and School 2, were conveniently selected. For each school, three mathematics teachers who were teaching Grade 5 were sent to attend PD. Six teachers participated in the study. T1, T2, and T3 were from School 1, and T4, T5, and T6 were from School 2. The reason for using convenient sampling was based on choosing the nearest individuals to serve as participants by continuing the process until the required sample size had been obtained of those who happened to be available and accessible at that particular time (Cohen et al., 2018, p. 218).

Design phase

The researcher conducted preliminary observations with the six mathematics teachers in accordance with their respective timetables. While visiting the teachers in their respective classrooms, the researcher observed the dominant TCT approach teaching practice. The researcher documented exactly the procedures being displayed by all the six mathematics teachers. The researcher further video-recorded the data and these data were referred to as Video Data Gathering step 1 (VDG-1). Based on the identification of this practical problem of use of the TCT strategy, an intervention was required to change the practices and the beliefs of the teachers from TCT to SCT.

Intervention phase

The researcher conducted a five-day training on SCT to introduce teachers to innovative teaching methods within the South Sudan school context. This article focused only on two aspects of SCT: PS and MD. Appendices A and B show parts of the training manual designed for PS and MD. Hence, the number of days required to complete the training on both PS and MD was two days per aspect. The participating teachers were encouraged to execute the SCT approach during the training.



SCT, student centred teaching; PD, profressional development; Pre-DBR, pre-design based research; Post-DBR, post-design based research.

FIGURE 1: An illustration of the design-based research professional development depicting parts of the process in this research.

Implementation phase

After the training, the researcher instructed the participating teachers to go back to their respective schools. The researcher followed up with the teachers to observe how they were implementing aspects of the SCT approach. The observations were conducted following the schedule per individual timetables as they were teaching in their respective classrooms. The researcher observed each of them and documented or detailed their practice of SCT. The researcher also collected video data and these data were referred to as Video Data Gathering step 2 (VDG-2).

Analysis phase

In this phase, the researcher again invited the teachers for one day of discussion and reflection on the application of this new approach of teaching in the South Sudanese context. The aim was to listen to their experiences and beliefs as they operationalised SCT in their respective classrooms. During this workshop, the teachers shared the difficulties they encountered in the process of implementing the SCT approach. After discussing what worked and what was challenging to implement, the researcher and teachers agreed to focus on some aspects of the SCT and the teachers went back to implement it again. The researcher followed them to their schools and observed how the teachers were implementing this SCT strategy.

Data analysis

All the collected data in the form of interviews, observation, field notes, and video (VDG-1 and VDG-2) were analysed using the content analysis method. Content analysis is 'a research method for the subjective interpretation of the content of text data through the systematic classification process of coding and identifying themes or patterns' (Hsieh & Shannon, 2005, p. 1278). There are three kinds of content analysis: conventional, directed, and summative (Hsieh & Shannon, 2005). Directed content analysis is guided by existing theory or prior research by identifying key concepts or variables as initial coding categories, while conventional content analysis is used when researchers try to avoid using preconceived categories; in summative content analysis, keywords are selected based on previous research or the researchers' interests (Cohen, 2018; Hsieh & Shannon, 2005).

The researcher employed conventional content analysis to analyse the data using codes, categories, or themes from the textual data available. This conventional content analysis is appropriate because the existing theory on textual data is limited to allowing the categories and names for categories to flow from the data. This includes reading all the data repeatedly to achieve immersion and obtain a sense of the whole, where the data are read word by word to derive codes, by first highlighting the exact words from the text that appear to capture key thoughts or concepts.

TABLE 1: Characteristics of student-centred versus teacher-centred teaching for the case of problem-solving and mathematical discourse.

The characteristics of student-centred teaching (SCT) (Stephan, 2014; Walters et al., 2014)	Teacher x
Problem-solving (PS) (Stephan, 2014)	
PS in teacher-centred teaching - Level 0 (PS-TCT-L0): The teacher used tasks or routine problems from the curricular materials and made sense of a problem situation, and the students were expected to reproduce or imitate it.	-
PS in teacher-centred teaching - Level 1 (PS-TCT-L1): The teacher modelled how to solve and make sense of a problem situation, and the students worked together or independently to create their own solutions.	-
PS in student-centred teaching - Level 2 (PS-SCT-L2): The students were posed open-ended problems being guided by the teacher, and were asked to create their own, personally meaningful solutions. Teacher was doing the problem-solving rather than the students.	-
PS in student-centred teaching - Level 3 (PS-SCT-L3): The students were posed problems without being guided by the teacher, and asked to create their own, personally meaningful solutions.	-
Mathematical discourse (MD) (Drageset, 2015; Hufford-Ackles et al., 2004; NCTM, 2014; Stephan, 2014)	
MD in teacher-centred teaching - Level 0 (MD-TCT-L0): Teacher was at the front of the room and dominated conversation. Teacher acted only as questioner and focused on the correctness. Or IRE (Initiation by the teacher, Response by the students and Evaluation by the teacher).	-
MD in teacher-centred teaching - Level 1 (MD-TCT-L1): Teacher encouraged the sharing of mathematics ideas and directed speaker to talk to the class, not to the teacher only. Teacher questions focused on student thinking and less on answers. Or IRIRE (Initiation by the teacher, Response by students, Initiation by the students, then Response by the students and Evaluation by the teacher).	-
MD in student-centred teaching - Level 2 (MD-SCT-L2): Teacher facilitated conversation between students and encouraged students to ask questions of one another. Teacher asked probing questions and facilitated some student-to-student talk. Or IRE (Initiation by the teacher, Response by the students and Evaluation by the teacher).	-
MD in student-centred teaching - Level 3 (MD-SCT-L3): Students carried the conversation themselves. Teacher only guided from the periphery of the conversation. Teacher waited for students to clarify thinking of others. Student-to-student talk was student initiated. Students asked questions and listened to responses. Teacher questions may still guide discourse. Or IRIRE (Initiation by the teacher, Response by students, Initiation by the students, then Response by the students and Evaluation by the teacher).	-

Table 1 provides the guiding template for analysing the data which summarises the details of the characteristics for SCT versus TCT for the case of PS and MD. This study followed directed content analysis. The data collected before, during, and after the intervention were analysed using the conventional content analysis procedure defined above. The checklist provided by Stephsn (2014) as presented in Table 1 was used as an analytical tool. The extended version of the checklist is provided in Appendix 1. The genuine reason for choosing these instruments was their simplicity. It provided maximum opportunity for the researcher to explore more information. Each characteristic had four levels.

Starting from one end with TCT-dominated practice, Level 0, to full implementation of SCT, Level 3. Level 0 and Level 3 coincided with the two ends ‘directed’ and ‘open’ inquiry according to Stephan (2014). In this work, two more levels were introduced to be able to register and discuss the shift in teaching practice by the teachers after the intervention, if any.

Trustworthiness

The researcher has considerably applied the trustworthiness orchestrated by the four criteria proposed by Lincoln and Guba (1985), which include credibility, transferability, dependability, and confirmability. Credibility refers to adequately representing the constructions of the social world under study. In this case, the researcher designed interview questions that encouraged the participants to participate in answering them and subsequently leading to a conclusion. These questions were related to activities such as recording, photographing, and documenting information that would help improve the credibility of the research findings.

Transferability refers to the extent to which the study hypothesis could be applied to another context. The researcher ensured that the study conducted on PD of mathematics teachers in South Sudan, specifically the case of SCT, could also be carried out in other locations or places. Dependability refers to the coherence of the internal process and how the researcher accounted for changing conditions in the phenomena. It entails the concept of reliability, in which the researcher used the same methods to obtain the same results. The same method was used: the application of DBR with the help of intervention on SCT while keeping a complete record of all phases of the research process. Confirmability explains to what extent the researcher admitted their bias (Bordens and Abbott, 2018). In this study, the researcher ensured that correct data from the participants were collected without influencing the nature of the data to accomplish the intended goals.

Findings

The findings of this study are discussed in relation to the research questions presented above. The data collected from the six primary mathematics teachers through interviews, observations, and the field notes are used in the analysis. The engagement of these teachers in the two private schools with the students in PS and MD to enhance SCT before, during, and after the intervention is presented below.

Before the Intervention

The result of the preliminary observation, with the six mathematics teachers within their time schedules, with the goal of understanding the practices of these teachers in connection to PS and MD, was conducted and was provided as follows. In both schools, all six teachers started their instruction by standing in front of the students and

Introducing the lesson they had prepared for the day. All six teachers were engaged in teaching the mathematical content of algebra and algebraic expressions as presented in the textbook. The teachers’ actions are summarised in Table 2. In all the classes, the students copied and imitated what their mathematics teachers wrote and solved on the blackboard. The students were seated in fixed rows facing their teachers, with their backs to one another in every classroom of the six teachers, as shown in Figure 1.

TABLE 2: Observation of the six teachers' practices before the intervention.

School	Teachers	Teachers' actions
School 1	T1	Invited four students – two boys and two girls – to come in front of the classroom and told the students that this was the concept of like terms.
	T2	Commented that they would be learning about algebra on that particular day and began to define what algebra is.
	T3	Requested seven students – two boys and five girls – to come in front to demonstrate the idea of like and unlike terms in algebra.
School 2	T4	Requested for a boy and a girl to go to the front and said if we considered these two students in term of metaphor in algebra they may represent unlike terms.
	T5	Began their lessons by asking the students what algebra was.
	T6	

Source: Photographs (a and b) were taken at Airport View Primary school in Torit, on the 06 June 2022 by Atari Anthony. Photographs (c and d) were taken at Our Lady of Holy Rosary Primary School in Torit on the 07 June 2022 by Ohide Ben.

FIGURE 2: Seating arrangements of the students before the intervention: In (a) students are copying what the teacher wrote on the board while in (b, c and d) all the students listen while the teacher explains concepts and procedures.

Problem-solving (PS) in a teacher-centred classroom

All six teachers copied the tasks directly from the Grade 5 textbook without any modification and presented and solved the tasks for the students. Four teachers presented the tasks entirely on the chalkboard for the students. These same teachers took the autonomy of solving the tasks instead of allowing the students to struggle to solve the given tasks. Two teachers (T3 and T4) solved the first task for their students and requested the students to come to the front and solve the remaining tasks. The problems the teachers were giving to the students could be solved with rote memory methods, requiring the application of algorithms and rules to arrive at the solution. Figure 2 provides a glimpse of the mathematical teaching practices, while Table 3 provides some of the actions of the teachers coded according to the PS framework provided in Table 1.

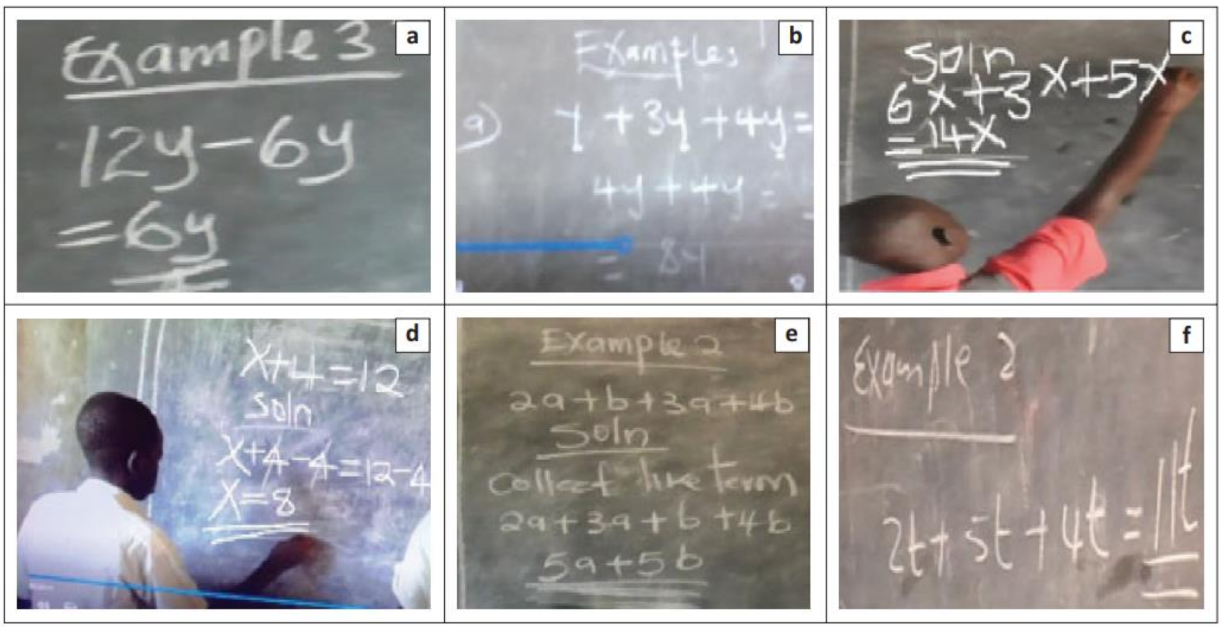
Mathematical discourse (MD) in a teacher-centred classroom

In both schools the teachers stood in front of the room and dominated conversation. Students were only to copy the correct answer. They both solved the task and instructed the students to reflect on the solutions in their notebooks. Some of these teachers tried to encourage the students to provide solutions but the students' response was focused on the teacher. For instance, T3 wrote the task on the blackboard and started solving it for the students by explaining to them what strategy to follow (MD-TCT-L0). The students were left with an option of writing down the solution. T4 engaged the class in a conversation. It is given as follows:

T4: Posed the task to students as y + 3y + 4y

Student 4-1: Solved it as 8y

T4: Why was 8y the solution?



Source: Photographs (a, b and c) were taken at Airport View Primary School in Torit on the 08 June 2022 by Opiaha Emmanuel. Photographs (d, e and f) were taken at Our Lady of Holy Rosary Primary School on the 10 June 2022 by Ohito Simon.

FIGURE 3: How the four teachers and two students solved the tasks: In (a), (b), (e) and (f) the teachers presented the tasks and solved them for their students; (c) and (d) each of the two teachers presented the task, and each of them invited a student to do the task on behalf of his or her colleagues.

Student 4-1: We added, y + 3y + 4y = 8y because these are like terms. [MD-TCT-L1]

The conversation between the T4 and the student ended there. T1, T2 and T6 followed a similar procedure to T4. Another episode from T5 class looks as follows:

T5: Presented the task: if a = 2, b = 5 and c = 4. Find the value of a+b+c.

T5: Started to solve the task to the class as a+b+c = 2+5+4 = 11.

T5 achieved this solution by talking to the students while in front of the classroom (MD-TCT-L0). Both teachers and students' workings are shown in Figure 3.

Teacher	Teachers' actions in a teacher-centred classroom	Coded according to the characteristics of problem-solving in teacher-centred teaching
T1	Selected and solved the task as ($12y$-$6y$ = $6y$), and told the students to copy correctly what he had solved on the chalkboard.	PS-TCT-L0
T2	Chose the task from the textbook and wrote as $y + 3y + 4y$ for the students to see. He later solved the task as $y + 3y + 4y = 8y$ and told the students this was how an algebraic task could be solved.	PS-TCT-L0
T3 and T4	Presented the tasks as $6x + 3x + 5x$, and invited a student to solve on behalf of the class. (T4 followed a similar procedure to T3).	PS-TCT-L1
T5 and T6	T5 and T6 used a similar procedure to T1 and T2.	PS-TCT-L0

TABLE 3: The six teachers' actions in connection to problem-solving before intervention.

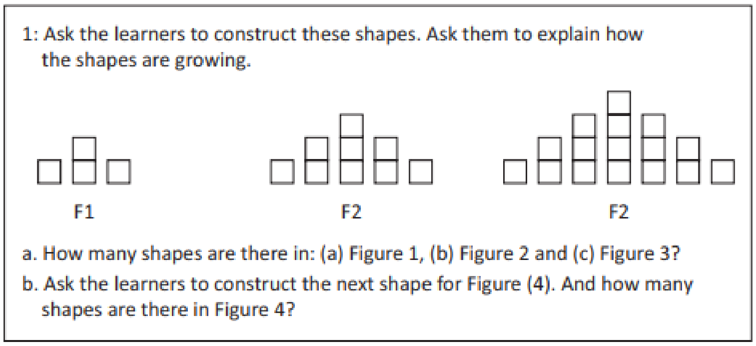
During the intervention

After designing lessons with tasks and activities that can facilitate SCT, the six teachers were invited to a four-day workshop, the contents of which were presented and discussed with experts in the field. Most of the materials were taken from recent research – informed teaching resources like NCTM (2014), Boaler (2016), YouCubed1 and others (see Appendix 2). The researcher introduced TCT and SCT during the workshop seminar and demonstrated them with examples. Among other characteristics of SCT, the teachers were exposed to the application of both PS and MD to enhance constructive mathematics teaching and learning. As shown in Appendix 2, different tasks were selected by the researcher for the training.

During the training, the researcher organised the six teachers into groups of two, making three groups of primary teachers. The researcher presented them with different tasks that could contribute to learning algebra, explicitly focusing on generalising patterns that can foster learning through PS and MD. The facilitator encouraged the teachers to discuss, interact, and come up with solutions for the given tasks. The teachers were asked to reflect on their experiences at the end of each day. The researcher-facilitator posed the following question: 'What features of teaching practice do you experience while teaching in your classroom?' The responses of the three groups were as follows (summarised by the researcher):

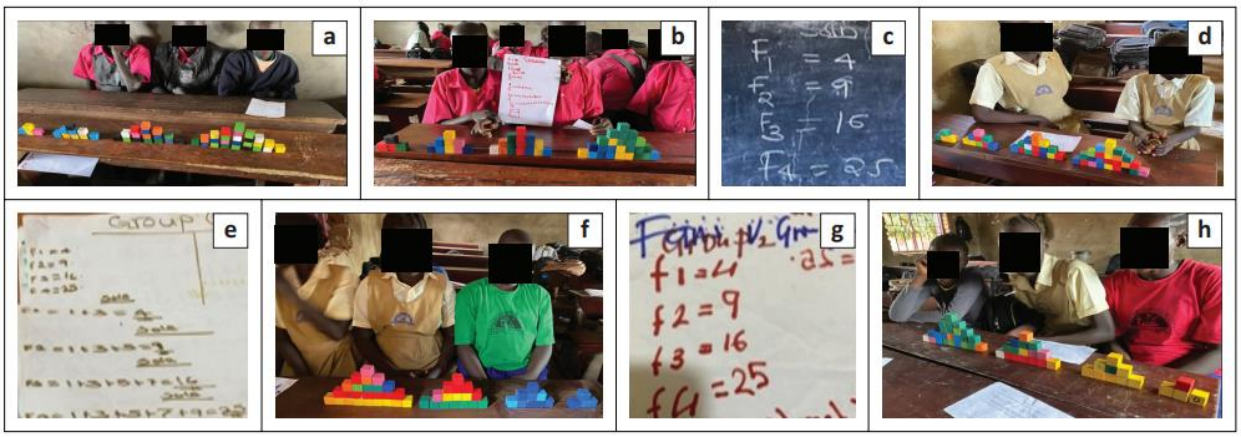
Group 1: The students rely only on what their teachers have given them. The teachers first solved the worked examples for the students and instructed them to do the other tasks individually. We realised that some students fear answering certain questions when asked. We also failed to use or design tasks that encourage mathematical discourse among the students, resulting in them doing the given classwork silently or without discussing it among themselves.

Group 2: We discovered that the students encountered significant difficulties in explaining, justifying, and clarifying their solutions. The teachers admitted that they have been implementing the TCT approach in their classrooms. We have failed to engage our students in group discussions; instead, we ask them to explain their solutions individually. Additionally, we always encourage our students to be seated in fixed rows.



Source: Adapted from Boaler, J. (2016). Mathematical mindsets [EPUB]. Jossey Bass Wiley.

FIGURE 4: Task based on a growing pattern of shapes used by teachers after Intervention.



Source: Photographs (a, b and c) were taken at Airport View Primary School in Torit on 20 June 2022 by Oturo Gamara. The photographs (d, e, f, g and h) were taken in Our Lady of Holy Rosary on the 22 June 2022 by Ameyu Andrew.

FIGURE 5: How the students presented their solutions in groups: In (a), (b), (d), (f), and (h) students showed the fourth figure using manipulatives while in (c), (e) and (g)provide the number of shapes in each figure.

Group 3: We are interested in solving every mathematical problem for our students and are fascinated by it. However, we have noticed that the tasks we design for our students discourage creativity and innovation, which in turn diminishes their sense of critical thinking. Additionally, we rarely use teaching aids or manipulatives in our classrooms. Moreover, we often encourage our students to master formulas or algorithms as the required strategy for solving mathematical tasks.

Post intervention

After the four-day workshop, the researcher visited again the six teachers in their respective schools. All the teachers were motivated to implement what they had learned. The results in connection to PS and MD are presented below.

Problem-solving (PS) in a student-centred classroom

After the SCT intervention, the teachers considered implementing some of the tasks they worked out together with other teachers during the training. One of the tasks (see Figure 4) was used by all the teachers in the two schools first. In Figure 5, one group from each of the six classrooms are shown where students are placed in groups, and they are engaged in solving problems. Furthermore, some selected parts of the data are presented in Table 4.

Mathematical discourse (MD) in a student-centred classroom

One shift in action that all the six teachers showed after the intervention was their decision to organise the students to sit in prespecified groups. T1 and T3 categorised the students into groups of six, T4 and T5 organised the students into groups of four, while T2 and T6 organised their students into prespecified groups of five and three respectively (see Figure 6). The teachers took the responsibility of moving from one group of students to another and inquiring of every group how they were doing the tasks.

Another task used by the teachers, with the researcher's assistance, was the so-called 'handshake problem' (see Task 4).

Classroom	Student-centred teaching implemented in classroom	Coded according to Table 1 (problem-solving in student-centred teaching)
T1's	The students started to discuss in groups and came up with the result as 4, 9, 16 and 25. T1 asked the students if there were any questions. Ojori: Why are you telling us to use the cubes? T1: Cubes can help you to understand how the shapes are growing.	PS-SCT-L2
T3's	T3 instructed the students to construct figures 1, 2, 3 and 4 and guided them to work in groups of 5. The students began to work and discuss in groups and presented the answer as 4, 9, 16 and 25. Then, T3 conducted the following conversation with one of the groups in the classroom: T3: Is there any comment? Oyito: Yes, the base for each of the figures was increasing by two for every odd number i.e. from base, 3 to 5 to 7 to 9 etc.	PS-SCT-L2
T4's	The students read and discussed the task in groups and got the result as 4, 9, 16 and 25. In one group, T4 had the following discourse: T4: What have you noticed from these figures? Kulang: The base for each of the figures was increasing by two on every odd number (i.e. Figure 1, with base 3 and 1 on top, Figure 2, with base 5, 3 and 1 on top, Figure 3, with base 7, 5, 3 and 1 on top and Figure 4, with base 9, 7, 5, 3 and 1 on top). T4: That is good, does someone have another way to explain it? *But the students kept quiet.*	PS-SCT-L2

TABLE 4: Example episodes evidencing problem-solving activity by the students in three classrooms.

In Appendix 2 and Figure 7). T1 told the students to sit in groups of six and discuss the given task. T1 moved from one group of students to another, asking them questions. The following discourse took place in T1's classroom:

Group 4: We discussed the task and came up with the solution as 5 + 4 + 3 + 2 + 1 + 0 = 15.

T1: How did you come up with this solution?

Atari: The 1st person greeted 5 people, 2nd person greeted 4 people, 3rd person greeted 3 people, 4th person greeted 2 people, 5th person greeted 1 person, and 6th person greeted 0 person.

T1: Why did you do that?

Atari: We added 5 + 4 + 3 + 2 + 1 + 0 = 15, and this is how we got.

T1: Okay…then?

T1: Do you guys have any questions for group 4?

Imoya: Why to say the 6th person will greet zero handshakes?

Atari: The 6th person will greet no body, hence zero handshakes.

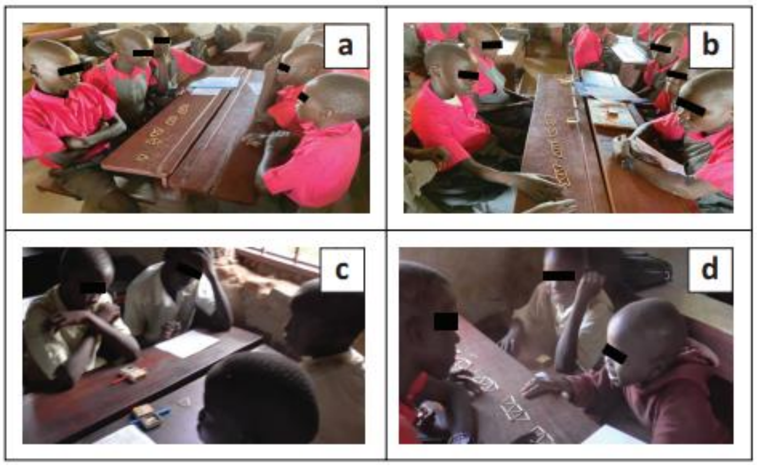
T1: Why not to say 6 people will greet 6 × 10 = 60 handshakes?

Ohide: Not like that sir, but it shall be like the way we solved it before.

T1: Okay, but are you sure?

Atari: Yes, we conversed among ourselves and we are thinking it is right.

T1: [To whole class] Is the solution given by group 4 right or wrong?



Source: Photographs (a, b and c) were taken at Airport View Primary School in Torit on 20 June 2022 by Oturo Gamara. Photographs (d, e, f, g and h) were taken in Our Lady of Holy Rosary on the 22 June 2022 by Ameyu Andrew.

FIGURE 6: Students seated in prespecified groups: In (a) and (b) students work in groups of 6 and in (c) and (d) students work in group of 4.

John: Maybe they are right.

T1: John how do you know that their answer is right?

John: Because of the way they are supporting their answer.

T1: Okay.

T1: Do you have any question for me?

None of the students asked any further questions.

T1 seemed to have shown how this task was to be done to his students before the researcher joined the classroom for observation. This was evident when his student, Atari, confidently solved the task without any difficulties. Therefore, I have reservations about how the student, Atari, presented the correct solution. Even during the training, the researcher presented this handshake problem to the six teachers, and none of the teachers was able to solve it correctly. Hence, this justifies the researcher's assertion.

T2 gave the task to the students and told them to work in a group of six. The students worked in groups as the teacher was inspecting the way they were working and asking them to explain their solutions. But Mary from group 5 wrote 20 + 10 = 30 handshakes.

T2 asked Mary to explain why she added 20 and 10 to get 30. Mary said: 'If 1 person greeted with 5 handshakes, then (4 × 5 = 20) and 2 × 5 = 10. Hence, the total handshakes would be 20 + 10 = 30 handshakes' (MD-TCT-L1). You may notice that T2 engaged the students to explain and justify their solutions despite their being wrong.

T3 instructed the students to solve the task in groups of six. The students in group 2 worked the task and presented solution as 6 × 5 = 30 handshakes. T3 asked group 2 to explain their solution:

Adaha: If 1 person has 5 fingers and there are 6 persons, then 6 × 5 = 30 handshakes.

T3: Do you have another solution different from this?

Source: The Photographs (a, b and c) were taken at Airport View Primary School in Torit on the 05 July 2022 by Oromo James. The photographs (d, e and f) were taken from Our Lady of Holy Rosary Primary School in Torit on the 07 July 2022 by Ohisa Ronald.

FIGURE 7: Seating arrangement and solutions presented by the students for the handshake problem: In (a), (b), (c), (d), (e) and (f), all the teachers organised their students to sit in the prespecified groups of six students per group

Students kept silent.

T3: Do you have any question for group 2?

Govind: How did you get 30 handshakes?

Adaha: We got by 6 × 5 = 30 handshakes.

T3: Adaha why not to say 4 × 6 = 24 handshakes?

Adaha: It could not be like that sir, instead you should multiply 6 × 5 = 30 handshakes.

In T3's classroom, it is also evident that the teachers elicited explanations from the students and it is notable that the students were confident in justifying their solution, even though it is not correct (MD-SCT-L2).

T4 instructed the students to do the handshakes problem in groups of six and the following narration was established:

T4: What is your solution?

Opiaha: We found 5 × 5 = 25 handshakes.

T4: How did you find 25 handshakes?

Opiaha: If 1 person made 5 handshakes then, 6 people would make (5 × 5) = 25.

T4: Is there a question?

Ojiok: Why did you say 5 × 5 = 25, instead of 5 × 6 = 30?

Opiaha: Because when you subtract (6 - 1) = 5, thus 5 × 5 = 25.

T4: What showed that this was your work?

Opiaha: We had talked in our groups.

T4: Do you have a question for me to answer?

Students kept silent.

Both T5 and T6 faced difficulties when implementing MD, and this was observed when the students failed to explain and clarify their solutions. T5 and T6 also failed to follow up on the students' reasoning regarding the solutions they found. The handshake problem posed a significant challenge for both the teachers and the students while solving it. The students explained and clarified their incorrect answers, although that was evidence of engagement from the teacher's side to implement MD in the classroom.

Discussion

The pre-intervention classroom observation in two private schools in South Sudan showed that the classrooms were dominated by TCT, a common trend across sub-Saharan African classrooms (Anyanwu & Iwuamadi, 2015; Bethell, 2016; Mueller et al., 2014; Mukuka et al., 2023). Further, this study focused on how the Grade 5 mathematics teachers were engaged in implementing the two characteristics of SCT, PS and MD, guided by Stephan's (2014) conceptual framework. The discussion is based on the data that were collected both pre and post interventions.

Problem-solving in student-centred teaching in South Sudan educational context

Before the intervention, the mathematics problems that the teachers used in the class were extracted from the Grade 5 textbook and presented to the students as tasks or examples. The nature of the problems that the teachers used did not engage the students in relational understanding (Skemp, 1978). Instead of investigating how solutions were determined, the students relied on the procedures or rules provided by the teachers to arrive at the solutions. The problems these teachers presented before the intervention were not inquiry-based tasks, as they failed to incorporate elements such as explanation, clarification, justification, and argumentation that could have deepened the students' understanding (Vygotsky, 1978). The problems used were routine, closed-ended tasks from the textbook that required the application of algorithms, procedures, and rules to find solutions (NCTM, 2014; Nurkaeti, 2018; Stephan, 2014).

Stephan (2014, p. 340) stressed that:

> … [in] a more directed approach, the teacher has modeled how to solve and make sense out of a problem situation, usually with a manipulative approach, and the students are working together or independently to create their solutions through discussion.

Most of the teachers in the two primary schools in South Sudan did that before the training on how to lead SCT. Teachers need to understand the difference between teaching via PS, about PS, and for PS (Schroeder & Lester, 1989; Van de Walle et al., 2020). Most of the routine and closed-ended tasks in the textbook might not provide an opportunity to implement teaching mathematics through PS, which can help students to do mathematics (Fosnot & Dolk, 2001; NCTM, 1989; Stephan, 2014).

After the intervention, most of the teachers demonstrated a more meaningful shift in their teaching practices and beliefs from PS-TCT-L0 and PS-TCT-L1 to PS-SCT-L2. Firstly, the teachers decided to use the open-ended tasks from the intervention (see Figure 3 and Appendix 2). They came to understand that not all tasks can provide opportunities for problem-solving. Closed-ended tasks are less rich than open-ended tasks (Boaler, 2016). Secondly, they allowed the students to struggle with the task instead of solving it for them. Thirdly, the teachers sought different strategies and approaches from the students. These strategies and techniques incorporated how the students were doing group work, posing mathematical tasks without guidance from the teachers, and creating their own meaningful solutions that the five teachers implemented. These approaches can help students to engage in mathematics (Fosnot & Dolk, 2001; NCTM, 1989; Stephan, 2014).

NCTM (2014) highlighted that a feature of PS is to engage the students in solving and discussing tasks that aim to boost mathematical reasoning and allow multiple entry points and varied solution strategies by the students. The six teachers' engagement in teaching through PS using open-ended tasks is promising. However, this shift observed in their mathematical practice might not be sustained unless they receive assistance through regular follow-up by experts, facilitators, or educators. The tasks teachers used during the observation post intervention were the tasks they learned during the workshop, which were limited to the concept of algebra, specifically generalisation from patterns of figures (Kaput & Blanton, 2008; Boaler, 2016). Such resources are needed if teachers are expected to develop their mathematical teaching practices.

Mathematical discourse in student-centred teaching in South Sudan educational context

We found that all six teachers were on level zero (MD-TCT-L0) before the intervention. The teachers were observed standing in front of the classroom, dominating the mathematical discourse, acting as questioners, and focusing on correctness. As shown in Table 4, the teachers initiate discussion, then the students respond, and then the teachers provide an assessment, terminating the discourse abruptly. NCTM (2014) highlights that students must also have opportunities to talk with, respond to, and question one another as part of the discourse community in ways that support the mathematics learning of all students in the class. This means that by talking, interacting, and discussing, the students are engaged in the construction of knowledge and skills that foster critical thinking and reasoning (Kaput et al., 2008).

After the intervention, three teachers (T1, T3, and T4) demonstrated a meaningful shift in their teaching practices and beliefs from MD-TCT-L0 to MD-SCT-L2. T2, T5, and T6 presented a somewhat lesser shift in their teaching practices and beliefs from MD-TCT-L0 to MD-TCT-L1. As shown from the discourses presented above, the teachers have followed not only the IRE model of communication (Drageset, 2015), but also others like IRIRE, several initiations (I) and responses (R) before providing feedback (E). T1, T3, and T4 were critically observed both facilitating conversation and encouraging the students to ask questions of one another, and the teachers asked probing questions that facilitated some student-to-student talk. T2, T5, and T6 were seen encouraging the students to share mathematical ideas, and the teachers' questions began to focus on students' thinking rather than the answers (Franke et al., 2009). The fact that these teachers have considered to provide more autonomy to their respective students is an encouraging development.

Implementing effective MD in a mathematical classroom is a demanding task. Demirci and Baki (2023) stressed that MD allows students to speak, think, and discuss mathematics, which involves explanation and debate on mathematical ideas. Hufford-Ackles et al.'s (2004) framework includes questioning, explaining mathematical thinking, using sources of mathematical ideas, and taking responsibility for learning. These primary teachers have agreed to provide more autonomy to their students in solving and discussing their strategies, primarily T1, T3, and T4. However, the remaining three teachers faced challenges in applying these developmental trajectories.

Another possible way to orchestrate productive classroom discussions is to implement the five elements of orchestrating mathematical discussions as described by Smith and Stein (2011). However, it is complex and demands interventions and investigations (Ballard, 2017; Faria et al., 2024; Larsson, 2015). Stephan (2014) emphasised that to guide the discussion, teachers need to not only accept both correct and incorrect solutions from the students but also purposely choose particular students' answers to begin the discussion and create debate in class. Smith and Stein (2011) called this process selecting and sequencing students' solutions so that students' mathematical reasoning and justification can build up with one another, providing a high level of student engagement and mathematical thinking.

Conclusion and Implications

The study aimed to engage primary mathematics teachers in two private schools in understanding the SCT approach, specifically in the cases of PS and MD. The findings revealed that initially, teachers were using tasks from the textbooks that were mostly routine, closed-ended tasks. Teachers were doing the mathematics while the students reproduced what their respective teachers did. The students were only expected to listen attentively to their teachers as the teachers explained concepts, standing in front of the blackboard, while the students faced them and listened. The teachers did not encourage the students to share their mathematical ideas or assign one of them to speak on behalf of their colleagues. Additionally, the teachers did not engage the students in conversations with each other or ask them questions. The teachers also failed to encourage the students to clarify their calculated answers. This shows that the mathematics practices of the six teachers in these two primary schools were dominated by the TCT approach (Bature, 2020; Stephan, 2014; Weimer, 2002).

After the intervention, the teachers' engagement in shifting their practices from TCT to SCT in connection to PS and MD was, somehow, remarkable. This was evidenced by the teachers' decision to use the tasks from the workshop for several reasons: first, the tasks aligned with their weekly plan for teaching algebra. Second, the open-ended tasks invited everyone to engage in the PS process. Finally, the teachers found the tasks interesting and used them in their classrooms. Rich tasks can allow students to engage in mathematics meaningfully (Boaler, 2016; NCTM, 2014). In connection to MD in classrooms, half of the teachers showed signs of engaging their students after the intervention. They demonstrated a more meaningful shift in their teaching practices and beliefs. However, the MD level was not implemented to the expected extent. This could be because these teachers were accustomed to traditional mathematics teaching and learning (Bature, 2020; NCTM, 2014; Stephan, 2014). There is a need to improve the intervention strategy by training the teachers for more than four days, as stipulated in the study. The study suggests that extending the intervention to a longer period is essential to achieve meaningful or satisfactory improvements. This will provide the teachers with enough time to be equipped with the features of SCT and boost their ability to implement the characteristics of PS and MD appropriately and successfully.

It should be noted that the shift in teachers' classroom practices was highly dependent on the intervention. Even the tasks the teachers used were adopted from the workshop. The teachers demonstrated that they were able to apply what was discussed at the workshops and apply their new knowledge in their own classrooms. Polly and Hannafin (2011) highlighted that 'in order to implement learner-centered pedagogies, teachers need extensive learning opportunities to acquire and internalize relevant knowledge and skills' (p. 120). Hence, more time would be needed to see if teachers' practices and beliefs shift from TCT to SCT in a long-lasting way. Stephan (2014) alluded that if teachers were able to apply both PS and MD in their daily teaching careers by attaining the maximum scale level, then there would be better improvement in the quality of education, leading to better performance among students. Furthermore, this better quality in education and performance could ultimately trigger a paradigm shift of teachers from the TCT to the SCT approach.

In general, professional learning that has characteristics such as being supportive, job-embedded, instructional-focused, collaborative, and ongoing is deemed effective (Hunzicker, 2011). This study, designed accordingly, demonstrated the possibility of developing PD to help teachers engage students and boost the process of learning mathematics effectively within their job context, teaching practice, and in line with the mathematical content (Soforon et al., 2023). Hence, training teachers to engage students in taking on the autonomy of both teaching and learning while the teachers act as facilitators in this process should be viewed from the broader context of professional learning. It demands resources, experts, and administrative support in general (Darling-Hammond, 2017; Haßler, 2020; Soforon et al., 2023).

To this end, DBR is employed in this study as part of the broader doctoral study. It allows us to contextualise the study within the two primary schools in South Sudan. As DBR is cyclic in nature, it implies further rounds of workshops for reflections and discussions with the teachers in connection to their practice in their respective classrooms. In this way, the teachers, and the researcher, as facilitators, can redesign tasks and activities to assist teachers in leading SCT. However, this study did not include the results of many rounds of such iterations, limiting the generalisation somewhat (Fowler et al., 2022).

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Appendix 1

TABLE 1-A1: TCT vs SCT PD (Directed versus Open).

The characteristics of student-centred teaching (Stephan, 2014; Walters et al., 2014)	Teacher x
Problem-solving (PS) (Stephan, 2014)	
PS in teacher-centred teaching - Level 0 (PS-TCT-L0): The teacher uses tasks or routine problems from the curricular materials and makes sense of a problem situation, and the students are expected to reproduce or imitate it.	-
PS in teacher-centred teaching - Level 1 (PS-TCT-L1): The teacher models how to solve and make sense of a problem situation, and the students work together or independently to create their own solutions.	-
PS in student-centred teaching - Level 3 (PS-SCT-L3): The students are posed open-ended problems guided by the teacher, and asked to create their own, personally meaningful solutions. Students are doing the problem-solving rather than the teacher.	-
PS in student-centred teaching - Level 3 (PS-SCT-L3): The students are posed problems without being guided by the teacher, and asked to create their own, personally meaningful solutions. Students are doing the problem-solving rather than the teacher.	-
Classroom Environment (CE) (Walters et al., 2014; Stephan, 2014)	
CE in teacher-centred teaching - Level 0 (CE-TCT-L0): The teacher has no focus on building respectful relationships and focuses on the solution of a problem.	-
CE in teacher-centred teaching - Level 1 (CE-TCT-L1): The teacher has some consideration of building respectful relationships and focuses on the solution of a problem.	-
CE in student-centred teaching - Level 2 (CE-SCT-L2): The teacher builds a respectful classroom environment and somehow encourages students in scaffolding, differentiation, and choice.	-
CE in student-centred teaching - Level 3 (CE-SCT-L3): The teacher builds a respectful classroom environment with focus on the individual (scaffolding, differentiation, and choice). Students are expected to (1) explain and justify their solutions and methods, (2) attempt to make sense of others' explanations, (3) indicate agreement or disagreement, and (4) ask clarifying questions.	-
Collaboration (CO) (Staples, 2008; Stephan, 2014)	-
CO in teacher-centred teaching - Level 0 (CO-TCT-L0): The teacher seems to ignore or avoid students working in groups and collaboration in problem-solving.	-
CO in teacher-centred teaching - Level 1 (CO-TCT-L1): The teacher considers groupwork but students tend to focus on individual effort.	-
CO in student-centred teaching - Level 2 (CO-SCT-L2): The teacher encourages students to pair with a partner or work with others in prespecified teams that range from two to six students.	-
CO in student-centred teaching - Level 3 (CO-SCT-L3): Teachers select appropriate tasks that allow all students access to the mathematics, use instructional strategies that prompt participation by all students, and support high-quality mathematics conversations within groups of two to six students.	-
Mathematical Discourse (MD) (NCTM, 2014; Hufford-Ackles et al., 2014; Stephan, 2014)	
MD in teacher-centred teaching - Level 0 (MD-TCT-L0): Teacher is at the front of the room and dominates conversation. Teacher is only questioner and focuses on correctness.	-
MD in teacher-centred teaching - Level 1 (MD-TCT-L1): Teacher encourages the sharing of mathematical ideas and directs speaker to talk to the class, not to the teacher only. Teacher questions begin to focus on student thinking and less on answers.	-
MD in student-centred teaching - Level 2 (MD-SCT-L2): Teacher facilitates conversation between students and encourages students to ask questions of one another. Teacher asks probing questions and facilitates some student-to-student talk. Or IRE (Initiation by the teacher, Response by the students and evaluation by the teacher).	-
MD in student-centred teaching - Level 3 (MD-SCT-L3): Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others. Student-to-student talk is student initiated. Students ask questions and listen to responses. Teacher questions may still guide discourse. Or IRIRE (Initiation by the teacher, Response by students, Initiation by the students, then Response by the students and Evaluation by the teacher).	-
Manipulatives or Tools (MT) (Bartolini & Martignone, 2020; Stephan, 2014)	
ML in teacher-centred teaching - Level 0 (ML-SCT-L0): Teachers do not use manipulatives or tools (concrete or virtual) in the teaching.	-
ML in teacher-centred teaching - Level 1 (ML-SCT-L1):' Teachers use manipulatives briefly without going deeper into the mathematical meanings.	-
ML in student-centred teaching - Level 2 (ML-SCT-L2): Teacher utilises tools, including manipulatives, notations, and symbols, as an integral part of teaching.	-
ML in student-centred teaching - Level 3 (ML-SCT-L3): Teacher utilises tools, including manipulatives, notations, and symbols, as an integral part of teaching. The teacher mediates mathematical meanings, using the artifact as a tool of semiotic mediation.	-

Note: Note: Please see the full reference list of the article, Soforon, O.G.B., Sikko, S.A., & Tesfamicael, S.A. (2024). Engaging primary mathematics teachers in two private schools in South Sudan: A case study on student-centred teaching in problem-solving and mathematical discourse. Pythagoras, 45(1), a775. https://doi.org/10.4102/pythagoras.v45i1.775, for more information.